OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4762

Mechanics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 11 January 2010
Morning

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 **(a)** An object P, with mass 6 kg and speed 1 m s^{-1}, is sliding on a smooth horizontal table. Object P explodes into two small parts, Q and R. Q has mass 4 kg and R has mass 2 kg and speed 4 m s^{-1} in the direction of motion of P before the explosion. This information is shown in Fig. 1.1.

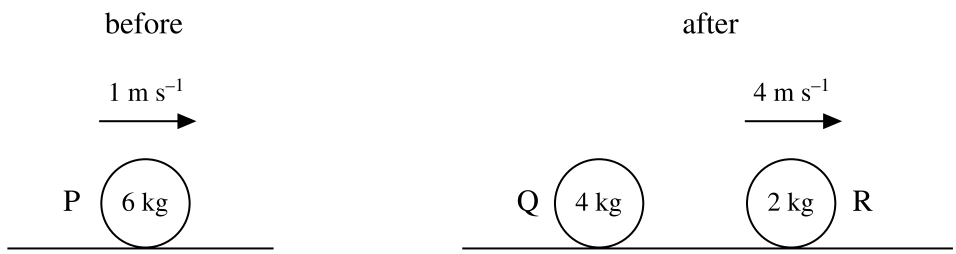

Fig. 1.1

(i) Calculate the velocity of Q. **[4]**

Just as object R reaches the edge of the table, it collides directly with a small object S of mass 3 kg that is travelling horizontally towards R with a speed of 1 m s^{-1}. This information is shown in Fig. 1.2. The coefficient of restitution in this collision is 0.1.

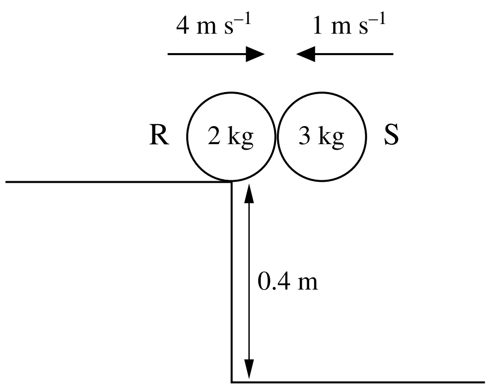

Fig. 1.2

(ii) Calculate the velocities of R and S immediately after the collision. **[6]**

The table is 0.4 m above a horizontal floor. After the collision, R and S have no contact with the table.

(iii) Calculate the distance apart of R and S when they reach the floor. **[3]**

(b) A particle of mass m kg bounces off a smooth horizontal plane. The components of velocity of the particle just before the impact are $u \text{ m s}^{-1}$ parallel to the plane and $v \text{ m s}^{-1}$ perpendicular to the plane. The coefficient of restitution is e.

Show that the mechanical energy lost in the impact is $\frac{1}{2}mv^2(1 - e^2)$ J. **[4]**

2 A car of mass 1200 kg travels along a road for two minutes during which time it rises a vertical distance of 60 m and does 1.8×10^6 J of work against the resistance to its motion. The speeds of the car at the start and at the end of the two minutes are the same.

(i) Calculate the average power developed over the two minutes. **[4]**

The car now travels along a straight level road at a steady speed of 18 m s^{-1} while developing constant power of 13.5 kW.

(ii) Calculate the resistance to the motion of the car.

How much work is done against the resistance when the car travels 200 m? **[5]**

While travelling at 18 m s^{-1}, the car starts to go **down** a slope inclined at $5°$ to the horizontal with the power removed and its brakes applied. The total resistance to its motion is now 1500 N.

(iii) Use an energy method to determine how far down the slope the car travels before its speed is halved. **[6]**

Suppose the car is travelling along a straight level road and developing power P W while travelling at $v \text{ m s}^{-1}$ with acceleration $a \text{ m s}^{-2}$ against a resistance of R N.

(iv) Show that $P = (R + 1200a)v$ and deduce that if P and R are constant then if a is not zero it cannot be constant. **[4]**

3 A side view of a free-standing kitchen cupboard on a horizontal floor is shown in Fig. 3.1. The cupboard consists of: a base CE; vertical ends BC and DE; an overhanging horizontal top AD. The dimensions, in metres, of the cupboard are shown in the figure. The cupboard and contents have a weight of 340 N and centre of mass at G.

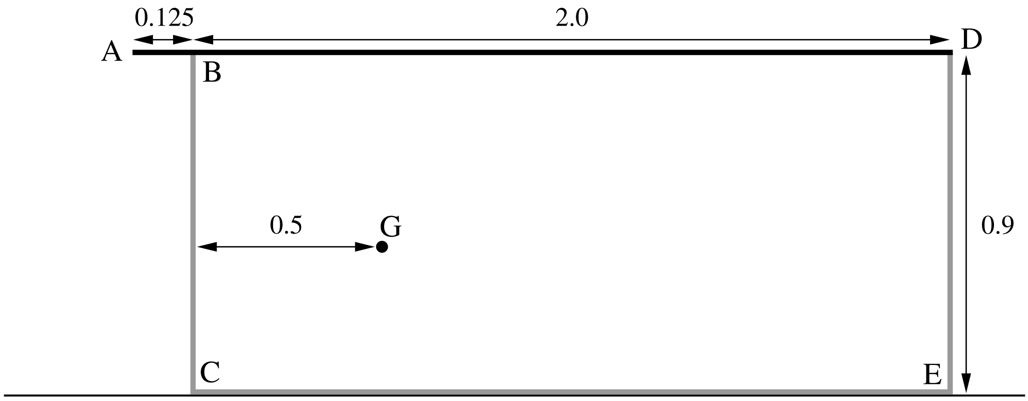

Fig. 3.1

(i) Calculate the magnitude of the vertical force required at A for the cupboard to be on the point of tipping in the cases where the force acts

(*A*) downwards, **[3]**

(*B*) upwards. **[3]**

A force of magnitude Q N is now applied at A at an angle of θ to AB, as shown in Fig. 3.2, where $\cos\theta = \frac{5}{13}$ (and $\sin\theta = \frac{12}{13}$).

Fig. 3.2

(ii) By considering the vertical and horizontal components of the force at A, show that the clockwise moment of this force about E is $\dfrac{30Q}{13}$ N m. **[3]**

With the force of magnitude Q N acting as shown in Fig. 3.2, the cupboard is in equilibrium and is on the point of tipping but not on the point of sliding.

(iii) Show that $Q = 221$ and that the coefficient of friction between the cupboard base and the floor must be greater than $\frac{5}{8}$. **[9]**

4 In this question, coordinates refer to the axes shown in the figures and the units are centimetres.

Fig. 4.1 shows a lamina KLMNOP shaded. The lamina is made from uniform material and has the dimensions shown.

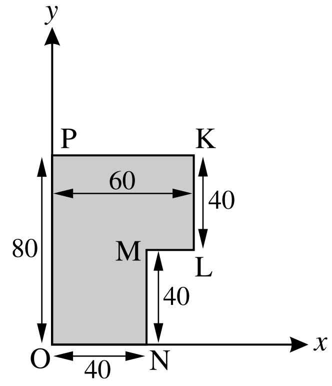

Fig. 4.1

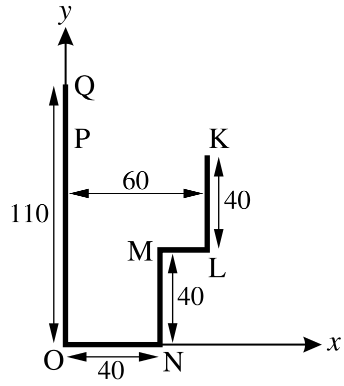

Fig. 4.2

(i) Show that the x-coordinate of the centre of mass of this lamina is 26 and calculate the y-coordinate. **[4]**

A uniform thin heavy wire KLMNOPQ is bent into the shape of part of the perimeter of the lamina KLMNOP with an extension of the side OP to Q, as shown in Fig. 4.2.

(ii) Show that the x-coordinate of the centre of mass of this wire is 23.2 and calculate the y-coordinate. **[5]**

The wire is freely suspended from Q and hangs in equilibrium.

(iii) Draw a diagram indicating the position of the centre of mass of the hanging wire and calculate the angle of QO with the vertical. **[4]**

A wall-mounted bin with an open top is shown in Fig. 4.3. The centre part has cross-section KLMNOPQ; the two ends are in the shape of the lamina KLMNOP.

The ends are made from the same uniform, thin material and each has a mass of 1.5 kg. The centre part is made from different uniform, thin material and has a total mass of 7 kg.

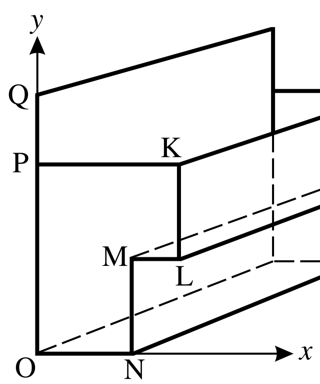

Fig. 4.3

(iv) Calculate the x- and y-coordinates of the centre of mass of the bin. **[5]**

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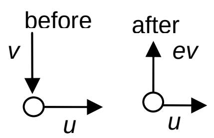
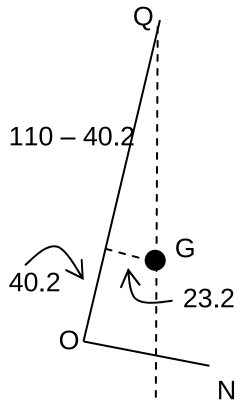
1 (a) (i)	Let vel of Q be $v \rightarrow$ $6 \times 1 = 4v + 2 \times 4$ $v = -0.5$ so 0.5 m s^{-1} in opposite direction to R	 M1 A1 A1 A1	 Use of PCLM Any form Direction must be made clear. Accept – 0.5 only if + ve direction clearly shown	4
(ii)	Let velocities after be R: $v_R \rightarrow$; S: $v_S \rightarrow$ PCLM +ve $\rightarrow$ $4 \times 2 - 1 \times 3 = 2v_R + 3v_S$ $2v_R + 3v_S = 5$ NEL +ve $\rightarrow$ $\frac{v_S - v_R}{-1-4} = -0.1$ so $v_S - v_R = 0.5$ Solving gives $v_R = 0.7 \rightarrow$ $v_S = 1.2 \rightarrow$	 M1 A1 M1 A1 A1 A1	 PCLM Any form NEL Any form Direction not required Direction not required Award cao for 1 vel and FT second	6
(iii)	R and S separate at 0.5 m s^{-1} Time to drop T given by $0.5 \times 9.8T^2 = 0.4$ so $T = \frac{2}{7}$ (0.28571…) so distance is $\frac{2}{7} \times 0.5 = \frac{1}{7}$ m (0.142857…m)	M1 B1 A1	FT **their** result above. Either from NEL or from difference in final velocities cao	3
(b)	 $u \rightarrow u$ $v \rightarrow (-)ev$ KE loss is $\frac{1}{2}m(u^2 + v^2) - \frac{1}{2}m(u^2 + e^2v^2)$ $= \frac{1}{2}mu^2 + \frac{1}{2}mv^2 - \frac{1}{2}mu^2 - \frac{1}{2}me^2v^2$ $= \frac{1}{2}mv^2(1 - e^2)$	 B1 B1 M1 E1	 Accept $v \rightarrow ev$ Attempt at difference of KEs Clear expansion and simplification of correct expression	
				4
				17

2(i)	GPE is 1200 × 9.8 × 60 = 705 600 Power is (705 600 + 1 800 000) ÷ 120 = 20 880 W = 20 900 W (3 s. f.)	B1 M1 B1 A1	Need not be evaluated power is WD ÷ time 120 s cao	4
(ii)	Using $P = Fv$. Let resistance be R N 13500 = 18F so F = 750 As v const, a = 0 so $F - R = 0$ Hence resistance is 750 N We require 750 × 200 = 150 000 J (= 150 kJ)	M1 A1 E1 M1 F1	Use of $P = Fv$. Needs some justification Use of WD = Fd or Pt FT **their** F	5
(iii)	$\frac{1}{2}\times 1200\times\left(9^2-18^2\right)$ $=1200\times 9.8\times x\sin 5-1500x$ Hence 145800 = 475.04846…x so x = 306.91… so 307 m (3 s, f,)	M1 B1 M1 A1 A1 A1	Use of W-E equation with 'x' 2 KE terms present GPE term with resolution GPE term correct All correct cao	6
(iv)	$P = Fv$ and N2L gives $F - R = 1200a$ Substituting gives $P = (R + 1200a)v$ If $a \neq 0$, v is not constant. But P and R are constant so a cannot be constant.	B1 B1 E1 E1	 Shown	
				4
				19
3 (i) (*A*)	Let force be P a.c. moments about C P × 0.125 – 340 × 0.5 = 0 P = 1360 so 1360 N	 M1 A1 A1	 Moments about C. All forces present. No extra forces. Distances correct cao	3
(i) (*B*)	Let force be P c.w. moments about E P × 2.125 – 340 × (2 – 0.5) = 0 P = 240 so 240 N	 M1 A1 A1	 Moments about E. All forces present. No extra forces. Distances correct cao	3

(ii)	$Q\sin\theta\times 2.125+Q\cos\theta\times 0.9$ $=\frac{25.5Q}{13}+\frac{4.5Q}{13}$ $=\frac{30Q}{13}$ so $\frac{30Q}{13}$ N m	M1 B1 E1	Moments expression. Accept $s\leftrightarrow c$. Correct trig ratios **or** lengths Shown	3
(iii)	We need $\frac{30Q}{13}=340\times 1.5$ so $Q = 221$ Let friction be F and normal reaction R Resolve $\rightarrow$ $221\cos\theta - F = 0$ so $F = 85$ Resolve $\uparrow$ $221\sin\theta + R = 340$ so $R = 136$ $F < \mu R$ as not on point of sliding so $85 < 136\mu$ so $\mu > \frac{5}{8}$	M1 E1 M1 A1 M1 A1 M1 A1 E1	Moments equn with all relevant forces Shown Accept $\le$ or = Accept $\le$. FT **their** F and R	
				9
				18
4 (i)	$4000\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=4800\begin{pmatrix}30\\40\end{pmatrix}-800\begin{pmatrix}50\\20\end{pmatrix}$ so $\bar{x}=26$ $\bar{y}=44$	M1 A1 E1 A1	Any complete method for c.m. Either one RHS term correct or one component of both RHS terms correct [SC 2 for correct $\bar{y}$ seen if M 0]	4
(ii)	$250\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$ $=110\begin{pmatrix}0\\55\end{pmatrix}+40\begin{pmatrix}20\\0\end{pmatrix}+40\begin{pmatrix}40\\20\end{pmatrix}+20\begin{pmatrix}50\\40\end{pmatrix}+40\begin{pmatrix}60\\60\end{pmatrix}$ $\bar{x}=23.2$ $\bar{y}=40.2$	M1 B1 B1 E1 A1	Any complete method for c.m. Any 2 edges correct mass and c.m. **or** any 4 edges correct with mass and x or y c.m. coordinate correct. At most one consistent error	5

(iii)		B1	Indicating c.m. vertically below Q	
	Angle is $\arctan\left(\frac{23.2}{110-40.2}\right)$	B1 M1	Clearly identifying correct angle (may be implied) and lengths Award for $\arctan\left(\frac{b}{a}\right)$ where b = 23.2 and a = 69.8 or 40.2 or where b = 69.8 or 40.2 and a = 23.2. Allow use of **their** value for y only.	
	= 18.3856…. so 18.4° (3 s. f.)	A1	cao	4
(iv)	$10\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 2\times1.5\times\begin{pmatrix}26\\44\end{pmatrix}+7\begin{pmatrix}23.2\\40.2\end{pmatrix}$	M1 B1 A1	Combining the parts using masses Using both ends All correct	
	$\bar{x} = 24.04$ so 24.0 (3 s.f.) $\bar{y} = 41.34$ so 41.3 (3 s.f.)	A1 F1	cao FT **their** y values only.	5
				18

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General Comments

Many excellent responses to this paper were seen and the majority of candidates attempted at least some part of every question and gained credit for their efforts. The standard of presentation was variable and, for some candidates, poor notation and failure to state the principles or processes being employed led to avoidable errors and to loss of marks. This was seen particularly in those parts of questions where a candidate was required to *explain* or *show* a given answer; many candidates did not structure a logical argument or give enough detail in either case.

Question 4 appeared to pose fewest difficulties to the majority of candidates.

Comments on Individual Questions

1 This was a high scoring question with the vast majority of candidates able to produce work worthy of significant credit.

(a) **(i)** This part posed few problems for the majority of candidates but many lost marks for failing to indicate the direction of the velocity they had found.

(ii) Almost all of the candidates showed understanding of the principle of conservation of momentum and Newton's experimental law and many obtained full marks for this part. Errors when they occurred were usually with signs. Those candidates who drew and labelled a diagram were on the whole more successful than those who omitted to do so.

(iii) This part caused difficulties for a minority of candidates. These, on the whole, did not appreciate that R and S would move as projectiles.

(b) Many candidates tried to treat kinetic energy as a vector and stated, incorrectly, that no energy would be lost in the horizontal direction. Others wrongly applied Newton's experimental law to both components of the initial velocity.

2 Many candidates seemed to understand the principles required to answer this question and applied them effectively.

(i) The majority of candidates gained full marks for this part. For those that did not, the most common error was to omit the term for gravitational potential energy. It was pleasing to see that the majority of candidates used units consistently and knew to take the time involved as 120 seconds.

(ii) Most candidates gained most of the marks for this part but only a small minority gained full marks by explaining clearly why the resistance to motion was equal to the driving force.

(iii) Most candidates obeyed the instruction in the question to employ an energy method. Some candidates found it difficult to correctly identify the term in gravitational potential energy; others omitted either one of the kinetic energy terms or the work done.

(iv) This part of the question was poorly done by quite a few candidates. Many used inconsistent notation and/or failed to provide sufficient detail to show the given answer. Many arguments to explain why the acceleration in this case could not be constant were muddled and incomplete. Some candidates tried to argue from the structure of the expression by rearranging it and did not relate the argument to the physical situation under investigation.

3 Many excellent answers were seen to this question but a significant minority of candidates struggled to provide sufficient detail when establishing a given answer.

(i) Almost all of the candidates obtained full marks for this part.

(ii) Most of the candidates understood that they needed to take moments for this part but failed to give enough detail in the working to show the given answer properly.

(iii) This part of the question was well done on the whole with the majority of candidates able to obtain correct values for the friction and the reaction forces. However, many failed to appreciate that an inequality was required and stated without any qualification that $F = \mu R$ and went on to say, again with no reason given, that $\mu > \frac{5}{8}$. Others understood that an inequality was required but did not appreciate the significance of the cupboard *not* sliding.

4 This was the highest scoring question on the paper with many excellent and completely correct answers.

(i) The majority of the candidates gained all the marks for this part.

(ii) Those candidates that were successful in part (i) were usually as successful in this part as well.

(iii) Many fully correct solutions were seen. Diagrams in many cases were good; those that were not were usually too small to be helpful and did not clearly show the centre of mass below the point of suspension.

(iv) Many candidates successfully obtained full marks for this part of the question. A minority, however, attempted to reinvent the wheel instead of using the results already established. Of those who were unsuccessful, many included a top to the bin or failed to appreciate that there were two ends to it.